9A) P162. Sadler

1) $2\cos^2\theta + 3 = 5 - 2\sin^2\theta$

$LHS - RHS = 2\cos^2\theta + 3 + 2\sin^2\theta - 5$

$= 2(\cos^2\theta + \sin^2\theta) - 2$

$= 2 - 2 = 0 \quad \therefore LHS = RHS \quad \square.$

2) $\sin\theta - \cos^2\theta = \sin\theta\,(1+\sin\theta) - 1$

$LHS - RHS = \sin\theta - \cos^2\theta - \sin\theta(1+\sin\theta) + 1$

$= \sin\theta - \cos^2\theta - \sin\theta - \sin^2\theta + 1$

$= -(\cos^2\theta + \sin^2\theta) + 1$

$= -1 + 1 = 0 \quad \therefore LHS = RHS$

3) $(\sin\theta + \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta. \quad \square.$

4) $RHS = (\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$

$= LHS$

5). $\sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \sin^2\theta + \cos^2\theta - 2\cos^2\theta$

$= 1 - 2\cos^2\theta.$

6) $LHS = \sin^4\theta - \sin^2\theta = \sin^2\theta\,(\sin^2\theta - 1)$

$= \sin^2\theta\,(\sin^2\theta - \sin^2\theta - \cos^2\theta)$

$= \sin^2\theta\,(-\cos^2\theta) = -\sin^2\theta\cos^2\theta.$

$RHS = \cos^4\theta - \cos^2\theta = \cos^2\theta\,(\cos^2\theta - 1)$

$= \cos^2\theta\,(\cos^2\theta - \sin^2\theta - \cos^2\theta)$

$= \cos^2\theta\,(-\sin^2\theta) = -\sin^2\theta\cos^2\theta. \quad \square$

7) left $= \sin^2\theta \tan^2\theta = \sin^2\theta \cdot \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\sin^4\theta}{\cos^2\theta}$

right $= \tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta} = \dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$

$= \dfrac{\sin^2\theta \cdot \sin^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^4\theta}{\cos^2\theta} =$ left. $\square$.

8) left $= 1 - \sin^2\theta$

$= \cos^2\theta$

right $= 1 + \cos^2\theta - 1$

$= \cos^2\theta$.

$\therefore$ LHS = RHS $\square$.

9) $\sin\theta\tan\theta + \cos\theta$

$= \sin \cdot \dfrac{\sin\theta}{\cos\theta} + \cos\theta$.

$= \dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} =$ right.

10) left $= \dfrac{1}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{1}{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} = \dfrac{1}{\dfrac{1}{\cos^2\theta}}$.

$= \cos^2\theta =$ right

$\square$.

11) left $= \dfrac{(\cos\theta+1)(\cos\theta+1)}{1-\cos^2\theta} = \dfrac{(\cos\theta+1)\cancel{(\cos\theta+1)}}{(1-\cos\theta)\cancel{(1+\cos\theta)}}$

$= \dfrac{1+\cos\theta}{1-\cos\theta}$ = right $\square$.

12) left $= \dfrac{\sin^2\theta - \cos\theta(1-\cos\theta)}{(1-\cos\theta)\sin\theta} = \dfrac{\sin^2\theta - \cos\theta + \cos^2\theta}{(1-\cos\theta)\sin\theta}$

$= \dfrac{\cancel{(1-\cos\theta)}}{\cancel{(1-\cos\theta)}(\sin\theta)} = \dfrac{1}{\sin\theta}$ = right.

3) left $= \dfrac{\sin^2\theta - \sin\theta\cos\theta}{-\cos^2\theta + \sin\theta\cos\theta} = \dfrac{\sin\theta\,(\sin\theta-\cos\theta)}{\cos\theta\,(\sin\theta-\cos\theta)} = \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$ = right. $\square$