

9A) P162.

Sadler

$$1) 2 \cos^2 \theta + 3 = 5 - 2 \sin^2 \theta$$

$$\begin{aligned} \text{LHS} - \text{RHS} &= 2 \cos^2 \theta + 3 + 2 \sin^2 \theta - 5 \\ &= 2(\cos^2 \theta + \sin^2 \theta) - 2 \\ &= 2 - 2 = 0 \quad \therefore \text{LHS} = \text{RHS} \quad \square. \end{aligned}$$

$$2) \sin \theta - \cos^2 \theta = \sin \theta (1 + \sin \theta) - 1$$

$$\begin{aligned} \text{LHS} - \text{RHS} &= \sin \theta - \cos^2 \theta - \sin \theta (1 + \sin \theta) + 1 \\ &= \sin \theta - \cos^2 \theta - \sin \theta - \sin^2 \theta + 1 \\ &= -(\cos^2 \theta + \sin^2 \theta) + 1 \\ &= -1 + 1 = 0 \quad \therefore \text{LHS} = \text{RHS} \end{aligned}$$

$$3) (\sin \theta + \cos \theta)^2$$

$$\begin{aligned} &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ &= 1 + 2 \sin \theta \cos \theta. \quad \square. \end{aligned}$$

$$\begin{aligned} 4) \quad \text{RHS} &= (\sin \theta - \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\ &= 1 - 2 \sin \theta \cos \theta \end{aligned}$$

$$\begin{aligned} 5) \quad \text{LHS} &= \sin^4 \theta - \cos^4 \theta \\ &= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) \\ &= \sin^2 \theta - \cos^2 \theta \\ &= \sin^2 \theta + \cos^2 \theta - 2 \cos^2 \theta \\ &= 1 - 2 \cos^2 \theta. \end{aligned}$$

$$\begin{aligned} 6) \quad \text{LHS} &= \sin^4 \theta - \sin^2 \theta = \sin^2 \theta (\sin^2 \theta - 1) \\ &= \sin^2 \theta (\sin^2 \theta - \sin^2 \theta - \cos^2 \theta) \\ &= \sin^2 \theta (-\cos^2 \theta) = -\sin^2 \theta \cos^2 \theta. \\ \text{RHS} &= \cos^4 \theta - \cos^2 \theta = \cos^2 \theta (\cos^2 \theta - 1) \\ &= \cos^2 \theta (\cos^2 \theta - \sin^2 \theta - \cos^2 \theta) \\ &= \cos^2 \theta (-\sin^2 \theta) = -\sin^2 \theta \cos^2 \theta. \quad \square. \end{aligned}$$

①

$$7) \sin^2 \theta \tan^2 \theta = \sin^2 \theta \cdot \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\sin^4 \theta}{\cos^2 \theta}$$

left =

$$\begin{aligned} \text{right} &= \tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} = \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta \cdot \sin^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^4 \theta}{\cos^2 \theta} = \text{left} \quad \square \end{aligned}$$

$$8) \text{ left} = 1 - \sin^2 \theta = \cos^2 \theta$$

$$\text{right} = 1 + \cos^2 \theta - 1 = \cos^2 \theta$$

\therefore LHS = RHS \square

$$\begin{aligned} 9) \sin \theta \tan \theta + \cos \theta &= \sin \theta \cdot \frac{\sin \theta}{\cos \theta} + \cos \theta \\ &= \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} = \text{right} \end{aligned}$$

$$\begin{aligned} 10) \text{ left} &= \frac{1}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \frac{1}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}} = \frac{1}{\frac{1}{\cos^2 \theta}} \\ &= \cos^2 \theta = \text{right} \end{aligned}$$

\square

$$1) \text{ left} = \frac{(\cos\theta + 1)(\cos\theta + 1)}{1 - \cos^2\theta} = \frac{(\cos\theta + 1)(\cancel{\cos\theta + 1})}{(1 - \cos\theta)(1 + \cancel{\cos\theta})}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta} = \text{right} \quad \square.$$

$$2) \text{ left} = \frac{\sin^2\theta - \cos\theta(1 - \cos\theta)}{(1 - \cos\theta)\sin\theta} = \frac{\sin^2\theta - \cos\theta + \cos^2\theta}{(1 - \cos\theta)\sin\theta}$$

$$= \frac{(1 - \cancel{\cos^2\theta})}{(1 - \cancel{\cos\theta})(\sin\theta)} = \frac{1}{\sin\theta} = \text{right}.$$

$$3) \text{ left} = \frac{\sin^2\theta - \sin\theta\cos\theta}{-\cos^2\theta + \sin\theta\cos\theta} = \frac{\sin\theta(\sin\theta - \cos\theta)}{\cos\theta(\sin\theta - \cos\theta)} = \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta = \text{right} \quad \square.$$